Individual homework

**Exercise 11**

Using the theorem of deduction and its reverse prove that:

7. ⊢ (¬q ∨ p) → ((s → q) → (s → p))

Step 1:

We can apply the reverse of the theorem of deduction like this:

if ⊢ (¬q ∨ p) → ((s → q) → (s → p))

then (¬q ∨ p) ⊢ (s → q) → (s → p)

then (¬q ∨ p), (s → q) ⊢ s → p

then (¬q ∨ p), (s → q), s ⊢ p

Step 2:

Using the definition of a deduction, we can generate the sequence (f1, f2, …, f5) for (¬q ∨ p), (s → q), s ⊢ p like this:

f1: ¬q ∨ p ≡ q → p - Hypothesis

f2: s → q - Hypothesis

f3: s

f3, f2 ⊢mp q:f4

f1,f4 ⊢mp p:f5

Step 3:

Using the deduction proved at Step 2, we can apply the theorem of deduction like this:

if (¬q ∨ p), (s → q), s ⊢ p

then (¬q ∨ p), (s → q) ⊢ s → p

then (¬q ∨ p) ⊢ (s → q) → (s → p)

then ⊢ (¬q ∨ p) → ((s → q) → (s → p)) (is a theorem)